Comments on "Three-Dimensional Finite, Boundary, and Hybrid Element Solutions of the Maxwell Equations for Lossy Dielectric Media"

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We would like to offer an observation on the interesting paper by Paulsen, Lynch, and Strohbehn.[1] Specifically, we would like to point out a potential source of error with the finite element analysis of the vector wave equation. The difficulty is perhaps better known in the context of vector eigenvalue problems, rather than in externally driven problems such as those considered by the authors.

When using finite element analysis to compute the resonant modes of a dielectric-loaded cavity, for example, it is well known that spurious modes are computed in addition to approximations to physical modes [1]. By the Fredholm alternative [2], the solution (if it exists) to an externally driven problem can be represented in terms of the eigenpairs of the corresponding eigenvalue problem. The unpleasant result is that if spurious modes exist in the finite element solution of the eigenvalue problem, the corresponding solution to the externally driven problem may consist of some linear combination of the nonphysical spurious modes in addition to the physical modes. In the worst case, there may not even exist a finite element solution to the driven problem, even when there is a clear physical basis for the existence of such a solution.

It must be acknowledged that our experience shows that nonphysical corruptions are not always present in externally driven problems. Mathematically, this is because the driving term often turns out to be orthogonal to the nonphysical modes. However, it is still quite easy to deliberately drive solutions that are entirely nonphysical [3]–[5].

It should also be noted that the problems considered in this paper have power loss due to both the finite conductivity within the finite element region and the integral equation on the finite element boundary. The resulting eigenvalues, if they were computed, would all have a substantial imaginary component due to the losses. The solution for any external source with a real frequency of excitation would then be guaranteed to exist. Indeed, as the excellent results presented by the authors suggest, the solutions to the externally driven problems may appear to be quite physical. Nevertheless, this should not be taken as proof that every excitation will yield a physical field in the finite element region. The fact still remains that the solution to the driven problem consists of a linear combination of both the physical and the nonphysical modes, whether or not these modes are explicitly computed. We therefore suggest that the authors address the issue of spurious corruptions in the finite element aspect of their method even though they are not directly solving the eigenvalue problem.

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[1]K. D. Paulsen, D. R. Lynch, and J W Strohbehn, *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 682–693, Apr. 1988.

Reply[2] by Daniel R. Lynch[3]

These comments address a very important topic, which is and ought to be the subject of continuing interest in the numerical analysis of EM problems. We have addressed the question of spurious modes in finite element solutions of TM problems earlier [6]. In that work we show that these modes do indeed arise, for example at mesh boundaries, but that they attenuate very rapidly (within three or four mesh spacings) with distance from their source. We have not studied these modes in the more general 3-D vector case, and look forward to the contribution cited in the Comment.

References

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[6] D. R. Lynch, K. D. Paulsen, and J. W. Strohbehn, "Finite element solution of Maxwell's equations for hyperthermia treatment planning," *J. Comput. Phys*, vol. 58, no. 2, pp. 246–269, 1985.

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Corrections to "Properties of Shielded Cylindrical Quasi-TE_{0nm}-Mode Dielectric Resonators"

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In the above paper[1] formula (6) should read:

$$\tau_f = C_L\alpha_L + C_b\alpha_b + C_a\alpha_a + C_h\alpha_h + C_{L1}\alpha_{L1} + C_{\epsilon r}\tau_{\epsilon r} + C_{\epsilon rs}\tau_{\epsilon rs}$$

and formula (11) should read:

$$A = \omega\mu_0 L/\left[2(2|C_L| + |C_{L1}|L/L_1 + |C_b|L/b)\right].$$

In addition, one number on the ordinate axis of fig. 4 is not appropriate. It should be 0.85 instead of 0.6. Also, in the sentence beginning on page 775 and ending on page 776, one symbol is missing. This sentence should read: "Usually the metal shield is made of homogeneous metal and the dielectric of the DR is isotropic so $\alpha_L = \alpha_b = \alpha_m$ and $\alpha_a = \alpha_h = \alpha_d$."

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[1]J. Krupka, *IEEE Trans Microwave Theory Tech*, vol 36, pp. 774–779, Apr. 1988.